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"A NEW THEORY OF WORKHARDENING"

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A NEW THEORY OF WORKHARDENING

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Abstract

The experimental evidence on the mechanical behavior of fcc, bcc and hcp metals is briefly reviewed. Extended linear hardening, largely independent of temperature, strain rate and other testing conditions, is found not only in all fcc metals (stage II) and in germanium but also in polycrystalline iron and simple steels. The average value of $K = G/\theta_{II}$ (shear modulus by work hardening coefficient in the linear range) is about 300 in fcc metals and 500 in steels. The corresponding value for hcp metals is by far larger, and it is concluded that the linear hardening in them cannot be compared to that in fcc and bcc metals.

A qualitative theory of easy glide is presented. Linear hardening in stage II is explained on the basis of three simple assumptions. The resultant theory is applicable to a great variety of materials, testing conditions and dislocation arrangements, and in particular also to the tangled dislocation structures which are believed to be due to interactions between point defects and dislocations. The expected rate at which energy is stored during glide, as well as slip line lengths, dislocation density, the role of intersection jogs and the Cottrell-Stokes law are discussed in the light of the theory, and good agreement with experimental fact is found.

The lower rate of linear work hardening in steels compared to that

in fcc metals is explained through the action of easy cross slip. A quite similar work hardening rate is occasionally observed in a linear range of stage III in pure fcc metals when, again, easy cross slip is believed to operate. Climb and "conservative climb" of dislocations can account for the remaining observations regarding stage III.

Introduction and Brief Survey of Experimental Evidence

The stress-strain curve of crystals of many different substances are qualitatively similar. No significant permanent plastic deformation takes place below the so-called "critical resolved shear stress". As the applied stress is raised beyond this level, yielding occurs with little or no work-hardening and sometimes even accompanied by a drop in stress. This stage is usually called the "easy glide" region, or "stage I". Straining beyond easy glide leads via a brief transition stage of rapdily increasing work hardening into "stage II", in which the work hardening coefficient is a constant, i.e. in which stress and strain are linearly related; also called the "linear hardening" range. Finally, in stage III, the workhardening coefficient diminishes again.

The above type of workhardening curve (Fig. 1) and in particular the range of linear workhardening in stage II has been the subject of numerous experimental as well as theoretical investigations, mainly in connection with pure fcc metals [1-16], but it is also found in fcc alloys [17-22] and, occasionally, in hexagonal metals [23-25], ionic crystals [26-28], and germanium [29], plus probably quite a many other substances.

Polycrystals do not usually exhibit easy glide, but a linear workhardening range comparable to stage II of fcc single crystals is often also found in them. Actually, the numerical value of Θ_{II}, the workhardening coefficient for single crystals in stage II, although varying by about the factor 2 for different orientations of the same material [4,5,7,9,30,31], is otherwise amazingly stable. In particular, θ_{II} apparently depends on the temperature only in the same way as the modulus of rigidity, G, so that $G/\theta_{II} = K$ is very nearly a constant [6,7,9,29,30,32-34]. Further, θ_{II} is almost independent of rate of straining [29,33], and composition of fcc alloys, except that it mostly seems to decrease somewhat with increasing alloying content (see for example refs. 5 and 21).

Investigations of linear hardening in bcc as well as fcc polycrystals shows that also their hardening coefficient is very reproducible, being little affected by temperature of testing [35], speed of testing [36], composition [35,36], prestrain [37], and grain size [38]. There is even adequate numerical agreement between $K = \theta_{II}/G$ measured for single crystals, and the value which one would deduce from the linear hardening range in polycrystals when converting the tensile stress and strain into shear stress and strain, according to some average orientation. For example, the linear hardening in polycrystalline brass is found between 50 kg/mm^2 and 60 kg/mm^2 tensile stress per 100% tensile strain. Since, for fcc metals, an average value for the conversion of tensile stress into shear stress, as well as of tensile strain into shear strain is 0.4 (see reference 39) one would obtain from the polycrystal curves the single crystal value $\theta_{II} \cong 55 \times (0.4)^2$ kg/mm^2 $\cong$ 9 kg/mm^2, which is consistent with the measurements of von Göler and Sachs [18].

In stark constrast to the discussed persistence of rapid linear hardening under a very wide variety of circumstances, "easy glide", not usually found in polycrystals, is so sensitive to many influences that it is often missed altogether. In fact, even small amounts of insoluble impurities erase it in nominally "pure" fcc metals, as was first recognized by Rosi[5] (see for example 2, 16, and 40). This is believed to be the reason why, in "pure" fcc metals, easy glide has been missed for many years[14], even in the course of otherwise very careful investigations as for example the early study by Karnop and Sachs[41] on glide in aluminum, and why easy glide has only been recognized as a general phenomenon in fcc metals since very pure metals have become available. In fcc solid solution alloys, on the other hand, easy glide has been known a long time, because its extent is greatly increased by the presence of soluble additions.

Easy glide is further dependent on specimen size[31,42], decreasing with increasing crystal diameter, and it may be influenced by surface treatments as well as clamping conditions (see 11 and 43). It is pronounced at low temperatures, but decreases with increasing temperature[6,16,29,32-34], and may vanish altogether at elevated temperatures. Above all, however, easy glide is prominent only for crystals oriented such that extensive single glide is to be expected, i.e. in crystals near the center of the standard triangle and towards the ⟨110⟩ corner, while crystals with their axis near ⟨111⟩ and ⟨100⟩ show little or no easy glide[2,4].

Actually, also stage I represents a range of linear hardening, but with a much smaller work hardening coefficient which, moreover, is strongly orientation dependent, increasing sharply towards ⟨100⟩ and ⟨111⟩ orientations.

The beginning of stage III as well as workhardening in this range are strongly temperature dependent. Stage III is usually thought to be caused by "dynamical recovery".

Theoretical Considerations on the Nature of Easy Glide

The yield stress of previously undeformed crystals is probably governed by their resistance against dislocation motion. This resistance is partly due to the so-called Peierls-Nabarro force, usually modified by the uncertainty of dislocation axes [44,45], is due to impurity and point defect locking of various types, to the resistance against the intersection of "forest" dislocations, the drag of the resultant intersection jogs, and/or due to the interaction between the glide dislocations and any other type of defect, plus the stress necessary to bow the free dislocation lengths out against the reaction of their line tension. But whatever may be the dominant effects, glide under the action of a slowly increasing stress cannot start uniformly throughout the crystal. Greater or lesser deviations from the calculated level of applied stress, as well as availability of dislocations, and local fluctuations of the total resistance to dislocation motion, must result in the initiation of plastic flow in certain areas while the rest of the material is still undeformed.

If a significant part of the initial flow stress is due to impurity or point defect locking, or if it is heightened by short or long range ordering [46-48], the dislocations which move first will experience a frictional stress which drops initially. In other cases the flow stress opposing the motion of the moving dislocations may rise slowly, but it is suggested, that only little hardening can take place until the crystal is filled with dislocations to the point that those moving in different directions (even though maybe on the same slip plane) mutually begin to block their paths, i.e. until there is no substantial region of the test length left in which no plastic deformation has taken place.

Sometimes, the lack of homogeneity in the early stages of deformation, the spread of dislocations into still undeformed regions, takes a very obvious form, as for example in the case of Lüders bands in iron or α-brass [49], or the spread of ordinary slip bands in ionic crystals [50,51], or Lüders bands observed in irradiated copper [52,53]. In agreement with the above suggestion that rapid hardening begins only when the specimen is filled with dislocations, "easy glide" in such cases apparently is terminated when the test length has been traversed by Luder's bands or is covered by slip lines. It is thus logical to correlate the easy glide region in general with slip taking place up to the point that the specimen has just attained a quasi-uniform dislocation distribution, before dislocations emanating from different centers, in which the yield stress was first exceeded, have begun mutually to block their progress.

Work Hardening in Stage I

Positive workhardening in stage I partly reflects the nonuniformity of the applied stress. If, due to grip effects for example, the applied force results in wide local stress fluctuations, yielding will first occur at stress peaks, where the yield stress is exceeded before the bulk of the material yields. The first regions to yield then workharden, and increasing proportions of the specimen are being stressed beyond the yield point. It is in general agreement with this type of geometrical hardening theory that Hauser and Jackson [54] can explain the orientation dependence of easy glide through grip effects.

In addition to the above effect, however, some true workhardening in easy glide of fcc metals, even under the most careful testing conditions, seems to take place. The lowest stress which will cause sustained motion of a dislocation in fcc metals at low and intermediate temperatures apparently rises continuously with the distance traversed already. If this was not so one could expect crystals to shear on individual planes; in extreme cases to fracture. Also one could not understand the temperature dependence of the workhardening rate in easy glide.

One theoretical reason for a true positive workhardening coefficient in easy glide was first given by van Bueren [55] : As a dislocation moves and intersects forest dislocations it acquires jogs which cause a drag on the dislocation line. In ideal single glide, the number of jogs formed is proportional to the area swept out by dislocation axes, which in turn is proportional to the shear deformation. Hence the resulting rate of work hardening depends on the relation between dislocation density and shear. If "unpredicted" slip takes place during easy glide, the density of forest dislocations increases with strain, giving rise to additional hardening. Interference of primary slip, by slip on secondary systems was investigated theoretically by Haasen and Leibfried [56,57] who accounted for the orientation dependence of the rate of workhardening in this way.

Another source of true workhardening in easy glide is that point defects are generated which interact with the dislocations to form tangles, and in this way hinder their progress [45] . Finally, "jogs" are created through the uncertainty of dislocation axes [45], which again cause a drag on the dislocations.

Although at this point no quantitative theory of workhardening during easy glide can be given, it seems safe to state (i) that the mechanisms mentioned above must operate (ii) that the linear hardening during easy glide, which is observed experimentally, is at least partly caused by them, and (iii) that these mechanisms should continue to operate even after stage II hardening has commenced. The three processes discussed are all temperature dependent and would contribute to τ_s, in Seeger's terminology [11,12].

On the Extent of Easy Glide

The dislocation density is always determined by the effective stress driving the dislocations forward, i.e. by the difference between applied stress and true frictional stress acting on the dislocations; but the shear deformation is equal to the product of Burgers vector, dislocation density, and mean free path. Since, for a given material, the effective stress during easy glide is clearly little dependent on straining conditions, the observed wide variations in the extent of easy glide, due to surface treatments, crystal orientation, temperature etc. must be due to correspondingly wide variations in the mean free dislocation path before stage II begins.

In the light of the preceding considerations, the main body of experimental evidence on the extent of easy glide becomes qualitatively understandable: Easy glide is terminated the earlier, the shorter the mean dislocation paths are. In orientations close to the symmetry line, or $\langle 111\rangle$ or $\langle 100\rangle$, in particular, slip will start practically simultaneously on intersecting systems, and dislocations migrating outward from different plastic regions will interact and block each others progress after very short free paths. Consequently, short easy glide near $\langle 100\rangle$, $\langle 111\rangle$, and orientations near the symmetry line of the standard triangle, is to be expected. The effect of insoluble impurities can also be understood on the basis of this concept, since dislocations will be held up by them and will rapidly multiply with correspondingly small mean dislocation paths.

Soluble impurities, by contrast, lower the stacking fault energy and give rise to dislocation locking as well as increased initial flow stress due to ordering phenomena. All of these effects, directly [38] or indirectly [45], cause the dislocations which travel outward from regions in which the yield stress is first exceeded to move long distances, i.e. cause long easy glide.

Specimens oriented for single slip and subject to bending stresses superimposed on tension will start to yield at the most highly stressed surfaces, causing dislocations on the same system and with the same Burgers vector to move towards the interior of the specimen. Although dislocations of the same slip system moving in the same direction interact slightly, mainly in the form of "glide polygonization" [58,59], they will not block each others motion as long as there is no strong obstacle in the path of the leading dislocations. Hence, the plastic regions spread from the surfaces inward and very long easy glide for this case is expected.

This effect is probably the explanation for the size effect (see also the discussion by Fleisher 16)), as well as for the very low work hardening observed by Röhm and Kochendörfer 49)) when shearing long single crystals of aluminum.

The absence of easy glide in polycrystals, except in cases where the onset of dislocation motion is coupled with a substantial drop in stress, as for example in iron, is in the first place a consequence of nonuniform stress distribution, since the resolved shear stress acting on the most favorably oriented slip systems in the different crystals varies widely. In addition, multiple slip begins very early in all crystallites of a polycrystal. In agreement with this thought is the long-known fact that the stress-strain curves of fcc polycrystals can be understood from the superimposition of the shear stress-shear strain curves of randomly oriented single crystals [41,61,62].

The influence of temperature on the extent of easy glide in pure fcc metals could not be understood until very recently. Now, however, it has been shown that dislocations in pure fcc metals "tangle" due to their interaction with point defects, in a process which has been named "mushrooming", [63,64]. It consists of a combination of glide and climb mechanisms, causing originally smooth glide dislocations to form irregular three-dimensional tangles. The dimensions of average tangles normal to the slip plane increase with increasing temperature. At very low temperatures "mushrooming" as such virtually ceases, although the "uncertainty of dislocation axes" still causes some slight movement of dislocations normal to their slip planes, even at liquid helium temperature [45]. The result of these processes is that the same amount of glide causes much more voluminous dislocation tangles at high than at low temperatures, and the slip which takes place until a quasi-uniform dislocation distribution is reached, at the end of easy glide, is consequently decreasing with increasing temperature.

It follows from the preceding arguments, and should be stressed, that the end of easy glide is not caused by the onset of secondary slip, as was first suggested by Röhm and Diehl [65], and has since been widely accepted. True, the onset of secondary glide will cause easy glide to end after only small additional shear, but easy glide will always stop when slip has spread quasi-uniformly through the material, whether or not secondary or unpredicted slip has speeded this process.

Experimental Evidence on Rapid Linear Hardening

According to the preceding arguments, the only feature common to all materials at the start of rapid linear hardening is that dislocations have been generated and been distributed throughout the specimens to form a quasi-uniform dislocation density. The actual dislocation *arrangements* at the onset of stage II, however, are widely different for different materials and testing conditions, as is known from experimental evidence. For example, pure fcc metals after deformation up to the start of stage II exhibit irregular dislocation tangles when the deformation took place at intermediate temperatures, but show little tangling and, instead, a profusion of long drawn-out prismatic loops when deformed at very low temperatures [45]. If "easy glide" - at a much higher stress level - took place after quenching or irradiation, the same metals show dense tangles, mostly aligned along active slip planes [64]. In fcc alloys of the α-brass type, by contrast, the dislocations are found in the form of sequences of pile-ups, held up behind obstacles [66-68], often represented by dislocations with different Burgers vectors. Qualitatively as well as quantitatively the latter dislocation arrangement agrees quite well with an earlier theory [67,69]. Moreover, in single crystals, slip up to the start of linear hardening is caused mostly by single glide, but in polycrystals much multiple slip must have taken place before linear hardening commences.

In spite of these wide variations in the dislocation arrangements, θ_{II}, the numerical value of the workhardening coefficient of stage II, is amazingly uniform. According to present best knowledge, all fcc single as well as polycrystals, even germanium, under any type of circumstances, exhibit a value of $G/\theta_{II} = K$, where G is the modulus of rigidity, between about 150 and several hundred, say, $G/\Theta_{II} = K \cong 300$ in average, within the factor 2 both ways. .

Even more astounding are the results on steels, accumulated by MacGregor and his group [35-37,70] which have been partly referred to already. From these it appears that a linear workhardening coefficient in tension is found for steels, which is persistent against every type of pretreatment or variation in testing conditions which have been tried. For practically all tests made and all steels investigated this workhardening coefficient equals 65 kg/mm tensile stress per unit tensile strain within a factor of less than 2 either way.

This is particularly surprising since large iron crystals show no range of rapid linear workhardening at all. The reason for this seems to be that one Luders band after the other passes through the specimens, and no quasi-uniform state of dislocation distribution is ever established.

Unfortunately, much less theoretical work has been done on the geometry of slip in bcc metals than is available for fcc metals. Therefore it is difficult to say what conversion factor should be used to transform the measured linear workhardening coefficient in tension into one corresponding to our Θ_{II}. However, in view of the multitude of slip planes as well as Burgers vectors available (assuming that in bcc polycrystals not only 1/2 <111> but also <100> act as Burgers vectors) one should think that the conversion factor from tensile stress to shear stress as well as from tensile strain to shear strain should be somewhat larger than the value 0.4 taken above for fcc metals, and be closer to the limiting value of 0.5. With the latter value, and with $G = 8000$ kg/mm^2 one obtains $K = G/\Theta_{II} \cong 8000/65 \times (0.5)^2 \cong 500$, a value somewhat above the average but still within the limits previously given for fcc metals.

The workhardening coefficient of crystals of hexagonal metals, on the other hand, is substantially smaller, amounting to about 0.7 kg/mm^2 for zinc at room temperature. This means that the previously introduced parameter, K, giving the ratio of workhardening coefficient to modulus of rigidity, would amount to several thousand. Moreover, as is also the case for iron, the polycrystal workhardening curve for hcp metals cannot be derived from those of the single crystals. It is therefore submitted that the linear range of workhardening sometimes found in hcp metal crystals is not comparable to stage II hardening in fcc metals. The reason for this may be that no quasi-uniform distribution of dislocations (which we believe is the prerequisite for stage II hardening)was established in the single crystalline hcp metal specimens investigated so far. This could ultimately be a consequence of the lack of interpenetrating slip systems.

Theory of Linear Hardening

From the foregoing it is evident that a general theory of stage II hardening should not depend on the magnitude of the Peierls stress, or of the stacking fault energy, nor on the presence of dislocation pile-ups, nor on the occurrence of multiple slip, since none of these factors seem specifically to influence the value of θ_{II}/G. It is also plain that long-range internal stresses cannot be responsible, because only an insignificant proportion of the plastic strain is removed on unloading. It is therefore attempted to base the theory of stage II hardening on no assumption, except that during linear hardening a quasi-uniform dislocation pattern exists which changes in its dimensions but remains similar to itself; may it consist of pile-ups or tangles or whatever:

Once the material is filled with dislocations, at the end of easy glide, all factors named before to determine the frictional resistance against dislocation motion remain in action much the same as in stage I, namely at about the level of the stress during easy glide. By contrast, the reaction of the dislocations against bowing out, which is due to their line tension, and which shall be designated by the symbol τ_l, is greatly affected. The reason for this is that the dislocations now start to hinder each others motion, mutually blocking or anchoring or pinning on parts, and leaving only segments of average length $\bar{l}$ free to move coherently. The stress required (over and above the total frictional stress due to all other causes) to bow out a dislocation of length l beyond its critical radius of curvature is about equal to Gb/l, or

$$\tau_l \cong Gb/\pi\bar{l} \qquad (1a)$$

since it is the longest coherent dislocation lengths which are being activated at any given moment.

In a quasi-uniform dislocation array of average free lengths $\bar{l}$ the distance between nearest dislocations is $c\bar{l}$, where c is a number not far from unity, say between 1 and 2. A loop emitted from a particularly long dislocation link will thus spread into a loop of radius $c\bar{l}$ before its different segments meet other dislocations, and it will spread to an average of $r_l = \alpha c \bar{l}$ if only the fraction α of all dislocations are positioned so that they could stop the spreading loop. A number dn of newly formed loops per unit volume will thus contribute a shear strain increment

$$d\gamma = dn\, b \pi r_{\ell}^{2} = dn\, b \pi (\alpha c \bar{\ell})^{2} \qquad (2)$$

Let, further, the dislocation density in the crystal be given by,

$$\rho = m/\bar{\ell}^{2} \qquad (3)$$

where m is a small number, somewhat depending on the actual dislocation distribution. In a regular three-dimensional network m is in the order of 3, a little higher, say m = 5, for a less regular arrangement. To this density the new loops add, but not to the full extent of the loops' circumference. Parts of the spreading loops encounter, and annihilate, dislocations of opposite Burgers vector, and thus not only vanish themselves but in addition eliminate those dislocation segments which were in their path. Other portions may react with dislocations in different system, particularly if multiple slip takes place. As a result, dn newly formed loops per unit volume add

$$d\rho = dn\, \beta\, 2\pi r_{\ell} \qquad (4)$$

to the dislocation density, where β is a number between 0 and 1. It is then

$$d\rho = -2m\, d\bar{\ell}/\bar{\ell}^{3} = 2\pi\beta r_{\ell}\, dn = 2\pi\beta\alpha c \bar{\ell}\, dn \qquad (5)$$

Using eq. (5) to express dn in terms of $d\bar{\ell}$ yields

$$dn = -m\, d\bar{\ell}/\pi\beta\alpha c \bar{\ell}^{4} \qquad (6)$$

and by inserting this into eq. (2) one obtains

$$d\gamma = -mb\alpha c\, d\bar{l}/\beta \bar{l}^2 \tag{7}$$

Finally, with eq. (1a) yielding

$$-d\bar{l}/\bar{l}^2 = \pi\, d\tau_l/Gb \tag{1b}$$

the connection between $d\tau_l$ and $d\gamma$ is found as

$$d\gamma = (\pi m\alpha c/G\beta)\, d\tau_l \tag{8}$$

This expression gives the increase of that part of the stress which is due to the line tension of the dislocations resisting the bowing-out of links, with the increase of shear deformation. Here G is the modulus of rigidity as before, while, to sum up, the other parameters were introduced as follows:

m connects the average free dislocation link length, $\bar{l}$, with the dislocation density, and is estimated at about 5.

α is the reciprocal of the fraction of dislocations, encountered by a spreading loop, which are oriented so that they can stop its progress.

c is a constant linking the average distance between dislocations on the slip plane to their average free length. It probably has a value not far from 3/2.

β is the extra dislocation length, expressed as a fraction of the circumference of an average newly formed loop, which is added to the dislocation content of the crystal if one new loop spreads out.

Since it is claimed that the increase in τ_ℓ is by far the most important contribution to workhardening in stage II, the workhardening coefficient θ_{II} will be close to $d\tau_\ell/d\gamma$, and the K factor, introduced before as $K = G/\theta_{II}$ is found as

$$K \cong \pi m c \alpha/\beta \approx 8\pi\alpha/\beta \cong G/\Theta_{II} \qquad (9)$$

This result is most intriguing. Note in particular that K does not depend on the Burgers vector. Therefore, within the limits that a pile-up of n dislocations can be regarded as one dislocation with a Burgers vector of n-fold strength, the above calculation can be applied to crystals containing dislocation pile-ups, as for example α-brass, just as well as to those containing tangles, say copper. The actual values of the parameters may vary somewhat from case to case.

In order to obtain a numerical value for K it is necessary to investigate the two important parameters α and β, all other parameters being known within fairly narrow limits.

Dislocations encountered by a spreading loop will only block its progress if they are favorably oriented. Mutually perpendicular dislocations will not interact much, except that a certain stress is required for the actual intersection (presumed to be small compared to τ_ℓ) and that intersection jogs are produced if the dislocations happen to have non-parallel Burgers vectors. In first approximation, it is only parallel or near-parallel dislocations which block mutually, and thus α may be taken as 1/3 for almost randomly oriented dislocations, and probably closer to 1/2 if the dislocations are mainly parallel to their own slip planes.

Of those encounters which cause blocking, one half, in single glide, will be between dislocations of opposite sign. These will not add to the effective dislocation content but in fact will reduce it. By how much, in average, depends on detailed circumstances, in particular how closely the two slip planes concerned are spaced and whether cross slip and climb takes place or not. In stage II these latter processes do not operate over appreciable distances, and the net reduction of effective* dislocation length due to encounters between a part of a spreading loop and dislocations of opposite sign may be estimated by assuming that one half of the encounters with opposite Burgers vectors will not give any net change, and one half will remove an equal length. Loop parts which encounter dislocations of like sign are assumed to be simply held up.

All in all, then, one half of an average loop will simply be blocked, one quarter will react to form pairs of about the same blocking strength as the dislocation originally present possessed, and the last quarter will meet a dislocation segments of opposite Burgers vector, leading to mutual annihilation and the net removal of a dislocation length equal to one quarter loop circumference. The parameter β will therefore be $\beta = 1/2 + 0 - 1/4 = 1/4$.

With the above estimates of $\alpha = 3$ (for almost randomly oriented dislocations, which is a good approximation for all cases in which dislocation tangles are formed) and $\beta = 1/4$, the value of K finally becomes

$$K \cong 8\pi\alpha/\beta \cong 300 \qquad (10)$$

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* A pair of parallel dislocations of opposite sign has only a stress field extending roughly to a radius equal to their distance of separation. Therefore close pairs of opposite signs are no effective barriers for further dislocations.

Quite clearly, all parameters appearing in the calculation could have been shifted somewhat one way or the other, and they have obviously been chosen so as to give numerical agreement with the experimental average value of K = 300 for fcc metals, which was quoted above. However, it would be difficult to account for K smaller than, say, 100 or larger than, say, 1000. Conversely, some flexibility is necessary since, in nature, K is not a universal constant but does vary more or less within those extreme limits.

Special Features of the Proposed Theory of Linear Hardening

1) Pinning Points Due to Mushrooming, or Precipitation, Radiation Damage, etc.

On superficial thought it appears as if the theory of mushrooming, encompassing strongly temperature dependent processes, and the temperature independence of stage II are mutually incompatible. This is not so, in fact.

Present best evidence indicates that dislocation tangling operates already at the smallest strains [45,71]. In this process, even during single glide, anchoring points are formed along the dislocation lines due to interactions with point defects as well as due to "jogs" formed through dislocation "uncertainty" [45]. In a previous paper [64] the dislocation behavior in mushrooming was considered qualitatively, but, regardless of detailed behavior patterns, easy glide will end only when a quasi-uniform dislocation density within the specimen has been reached.

In stage II, in the presence of additional strong pinning, the dislocations bow out between adjacent pinning points, just as discussed before, and, again, the stress difference between the applied resolved shear stress and the total frictional stress on the dislocations must be inversely proportional to the average length between adjoining pinning points. The only major difference now is that the pinning points are only partly, and not exclusively due to the mutual blocking action of the dislocations.

Let, then, the average free length between pinning points due to the presence of other dislocations be denoted by the symbol $\bar{\ell}$, as before, while the average distance between pinning points due to all other causes is ℓ_o. Then there are, on a total length, L, of dislocation line, $L/\bar{\ell}$ and L/ℓ_o pinning points due to dislocations and due to other causes respectively. The total number of pinning points on length L is thus $L\,(1/\bar{\ell} + 1/\ell_o)$, the average free dislocation length becomes $\lambda = \bar{\ell}\,\ell_o/(\bar{\ell} + \ell_o)$, and the stress necessary to overcome the line tension of the longest free lengths is now

$$\tau_\lambda = Gb/\pi\lambda \qquad (1c)$$

From here on there are no important differences between the considerations given above to account for stage II hardening and the present case of additional pinning points: As long as ℓ_o remains constant, workhardening in stage II is again caused by the blocking of moving dislocations where they encounter near-parallel dislocation links, limiting the radius of the average expanding loop to $r_\ell = \alpha c \bar{\ell}$, and giving rise to the shear increment $d\gamma = dn\, b\, \pi\, (\alpha c \bar{\ell})^2$ if dn expanding loops have formed per unit volume. Again, just as before, the dislocation density is given by $\rho = m/\bar{\ell}^2$ and, in the same way as for normal stage II hardening, one obtains $d\gamma = -\,mb\alpha c d\bar{\ell}/\beta\bar{\ell}^2$ (eq. 7). The corresponding increase in stress, however, is given by

$$d\tau_\lambda = -\,(Gb/\pi)\, d\lambda/\lambda^2 \qquad (1d)$$

but since, $1/\lambda = 1/\bar{\ell} + 1/\ell_o$ one obtains $d\lambda/\lambda^2 = d\bar{\ell}/\bar{\ell}^2 + d\ell_o/\ell_o^2$. As long as ℓ_o either remains constant, or remains proportional to $\bar{\ell}$ it is $d\lambda/\lambda^2 = d\ell/\bar{\ell}^2$, respectively $d\lambda/\lambda^2 = B d\bar{\ell}/\bar{\ell}^2$ with B a proportionality constant. Hence, $d\tau_\lambda = B\, d\tau_\ell$ and $d\gamma = B\,(\pi m \alpha c/G\beta)\, d\tau_\ell$, i.e. the workhardening coefficient is not affected (beyond possible changes in the geometrical parameters) by additional pinning points of constant density; and is only increased by the factor $B = 1 + (\bar{\ell}/\ell_0)^2 \frac{d\ell_0}{d\bar{\ell}} = 1 + e$, if ℓ_0 remains proportional to $\bar{\ell}$ as $\bar{\ell} = e\ell_0$. The above calculation applies equally to all cases of additional pinning, be it due to mushrooming, precipitates, debris left by radiation damage, or any other cause.

The above considerations can be put into different words and generalized as follows: The stress increase on a change of λ, the distance between adjacent pinning points, is given as

$$d\tau_\lambda = -(Gb/\pi)\, d\lambda/\lambda^2 = -(Gb/\pi)\,(d\bar{\ell}/\bar{\ell}^2 + d\ell_0/\ell_0^2) = d\tau_\ell + d\tau_s \qquad (11)$$

where $\tau_\ell = Gb/\pi\bar{\ell}$ and $\tau_s = (Gb/\pi\ell_0 + \text{const})$, i.e. it is simply the sum of the stress increase due to the change of $\bar{\ell}$ alone and that due to the change of ℓ_0 alone. Moreover, if the extra pinning points give way before the dislocation can fully bow out between them, they only add to the overall frictional stress, τ_s in Seeger's terminology [11,12]. As long as τ_s either remains constant or rises linearly with the shear strain, linear workhardening in stage II will be observed. The workhardening rate which follows from the increase in τ_ℓ alone is θ_{II} as derived above.

2) Stored Energy

If dn dislocation links per unit volume bow out to form loops, the work thereby done on the specimen by the total applied stress, τ, is

$$dW_i = \tau\, d\gamma = \tau\, dn\, b\, \pi\, r_\ell^2 \qquad (12)$$

per unit volume. The work stored, per unit volume, is

$$dW_s = dn\,\beta\,2\pi r_l U \tag{13}$$

where U is the energy per unit length of dislocation line. In first approximation, for low dislocation densities, $U = Gb^2$, but dropping somewhat as the dislocation density increases, say, to a lower value of $U = Gb^2/\pi$.

With an average value of $U = Gb^2/2$, and neglecting the frictional stress on the dislocation by equating $\tau = \tau_\lambda = Gb/\pi\lambda$, the ratio of stored energy to work input becomes

$$dW_s/dW_i = \pi\beta\lambda/c\alpha\bar{l} \cong 2\beta\lambda/\alpha\bar{l} \tag{14a}$$

Introducing $G/\theta_{II} = K \cong 8\pi\alpha/\beta \cong 300$ from Eq. (10) above it remains

$$dW_s/dW_i \cong (16\pi/K)(\lambda/\bar{l}) \cong 16(\lambda/\bar{l})\,\% \tag{14b}$$

For "soft" metals, i.e. metals in which $\lambda = \bar{l}$ and in which the frictional stress on the dislocations is small, up to about 16% of the energy input is thus stored according to the present theory, less for others. So far, the contribution made to stored energy by the generation of point defects has been neglected. If point defect generation and the frictional stress on dislocations are also taken into account, the value of dW_s/dW_i is slightly reduced since point defect generation probably accounts for much but not all of the frictional stress. The stored energy has also been overestimated for another reason: As the dislocation density increases, not only the line energy for newly formed dislocations decreases, but the line energy per unit length of the total dislocation content. All in all, then, we arrive at the result that only several percent, and up to about one sixth of the work done during plastic deformation is retained as stored energy. This is in agreement with experimental evidence [72].

3) Mean Free Path and Slip Line Lengths

If the movement of several dislocations takes place cooperatively, in particular if pile-ups are formed, the workhardening rate in stage II is about the same as for the independent motion of single dislocations, because of the feature pointed out already, that the Burgers vector does not appear in the expression for θ_{II}. Hence, to the extent that pile-ups can be approximated to super-dislocations, the theory remains almost unaffected. Average dislocation paths, however, are then not a few times the distance between neighboring dislocations, but a few times that between neighboring pile-ups. At the same time, the relative displacement of the two sides of a slip plane which is caused by an expanding super-loop of piled-up dislocations is equal to the sum of their Burgers vectors. Slip lines and slip bands will thus arise. Since the workhardening rate is the same in either case one may understand why slip bands and "elementary structure" [73,74], i.e. slip in the form of somewhat coordinated and almost completely uncoordinated dislocation motion, so often exists side by side in the same specimen.

In the framework of the present theory, the slip line length, i.e. the final diameter of an expanding loop (or pile-up, as the case may be) is given by

$$\Lambda = 2 r_\ell^* = 2\alpha c \bar{\ell}^* \cong \pi G n b \alpha c/\tau_\ell \qquad (15)$$

The asterisks * have been used to direct the attention of the reader to the fact that groups of n dislocations may move cooperatively, whose effective Burgers vector is $b^* = nb$, and whose average free lengths ℓ^* are connected with τ_ℓ, the difference between the applied stress τ and the frictional stress, as $\tau_\ell = G b^* / \pi \ell^*$.

In first approximation, the frictional stress may be taken equal to the stress level during easy glide, τ_o, i.e. $\tau_\ell \cong \tau - \tau_o$. Since experimentally, in stage II, $\tau \cong \tau_o + (G/K)(\gamma - \gamma_o)$ as indicated in Fig. 1, and, therefore, $\tau_\ell \cong (G/K)(\gamma - \gamma_o)$ one obtains

$$\Lambda \cong \pi K n b \alpha c/(\gamma - \gamma_o) \tag{16}$$

and with $K = \pi m \alpha c/\beta = 300$, $\alpha \cong 3$, $c \cong 3/2$ and $b \cong 2.8 \times 10^{-8}$ cm this is

$$\Lambda \cong [n/(\gamma - \gamma_o)] \times 10^{-4} \text{ cm} \tag{16a}$$

Experimental determinations of slip line lengths [9,53,75,76] do indeed yield a relationship of the form $\Lambda = A/(\gamma - \gamma_o)$, with the proportionality constant for copper found as $A = 6 \times 10^{-4}$ cm, plus minus 30% or so. This indicates (as also does much experimental evidence), that in copper no proper pile-ups are formed, but that n is a number about 5, -and presumably less in average since only the longer more prominent lines have been measured. ¶These prominent lines cannot themselves have been due to the emission of just five consecutive dislocations. Several or many, but not intimately connected processes of loop formation must have taken place in close proximity. It is suggested that the explanation for such repeated action is similar to the one put forward in an earlier paper [77], namely that the annihilation of a link due to part of an expanding loop removes an obstacle for a neighboring link to act, and so on.

In this same paper [77] measurements on the average length of elementary lines in aluminum single crystals are quoted, together with the resolved shear stress which had been applied. The product $\tau\Lambda$ in these measurements is apparently a constant of average value $\tau\Lambda = 11.6 \times 10^4$ cm dynes/cm^2. This may be compared with the equation (15) above. If τ_o is neglected in comparsion to τ_ℓ, if n is taken equal to one, (since dislocations in aluminum do not form obvious pile-ups), if, further, the appropriate values for G and b are inserted, and if a and c are again taken as 3 and 1.5 respectively, one obtains $\tau\Lambda = 10.8 \times 10^4$ cm dynes/cm^2.

The close numerical agreement between the theoretical formula and these old measurements is to some extent fortuitous, but it does strengthen the confidence in the quoted measurements, and also provides a simple explanation for elementary lines, namely extremely fine cracks formed in oxide layers where single dislocations have moved. These would not tangle, as was pointed out repeatedly, [45,63,64] because point defects which could interact with them leave through the free surface.

4) Dislocation Density

The dislocation density in the present theory is given by $\rho = m/\bar{\ell}^2$ with m about 5. With $\tau \cong \tau_o + \tau_\ell \cong \tau_o + Gb/\pi\bar{\ell}$ and $\tau - \tau_o = (\gamma - \gamma_o)G/K$ this yields

$$\rho = m\left[\pi(\gamma-\gamma_o)/bK\right]^2 \cong 5\times 10^{11}(\gamma-\gamma_o)^2 \qquad (17a)$$

or $$\rho \gtrsim m\left[\pi(\tau-\tau_0)/Gb\right]^2 \qquad (17b)$$

Although these expressions yield values of approximately right magnitude for ρ, it seems that the dependence of dislocation density on stress and strain is mostly found differently, namely rising more slowly,[78-81]. Only Bailey and Hirsch[82] found a parabolic relationship of the predicted form for polycrystalline silver, namely $\rho \gtrsim 0.6 \times 10^{-5}\tau^2$ (equation (17b) yields $\rho = 0.8\times10^{-6}(\tau-\tau_0)^2$ for silver). One of the reasons why measurements usually give too low dislocation densities at higher strains probably lies in the fact, mentioned already before, that the energy of unit length of dislocation line drops with increasing dislocation density.

5) The Role of Intersection Jogs

When single crystals of pure fcc metals are strained, so that they change from single slip to double glide within stage II, no break is observed in the workhardening curve. In the framework of the present theory there would be scope to account for differences of K between macroscopic single and double glide, inasmuch as the parameters α or β or both could change. However, there is no a priori principle to indicate in which direction the change should go. The factor α should be little changed by multiple slip because a dislocation will be blocked in its progress if it counters another one more or less parallel to itself, regardless of the direction of the latters Burgers vector; either annihilating it, or reacting with it to give a dislocation on a different slip plane, or just being blocked by it. The parameter β, on the other hand, is affected in two opposing directions:

Encounters in which the two dislocation segments annihilate altogether become proportionately fewer in multiple glide, but encounters in which the dislocations get simply blocked also become fewer. Instead more dislocation reactions take place in which one new dislocation is formed from two old ones. All in all, the increase in β because of the reduction in dislocation annihilations would, for equal numbers of occurrences, more than counterbalance the decrease due to fewer simple blockings, but, in second approximation, it must be borne in mind that mutually repulsive dislocations on intersecting systems can bypass each other more readily than those on parallel slip planes. Since thus it is not clear in which direction θ_{II} should change, the conclusion is close at hand that the change cannot be drastic.

Nonetheless, the absence of any kink or discontinuity in the work-hardening curve of pure fcc metals to indicate the beginning of double glide is somewhat surprising. After all, intersection jogs must be formed, and they are known to cause a drag on dislocations, (see for example 45, 83, 84). Still, there are a few reasons which may account for this fact: (i) Like single slip, at the beginning of plastic deformation, double glide in pure fcc metals does not start discontinuously. (ii) Mushrooming as well as the uncertainty of dislocation axes cause a high density of "jogs" even in the absence of intersecting dislocations, and also remove them in statistical equilibrium [45] so that there may be no permanent change in jog density. (iii) The frictional stress due to all causes, including the jogs caused by mushrooming, is only in the order of τ_o, i.e. usually small compared to the stress at which double glide starts. Therefore the extra hardening effect of the intersection jogs, which presumably is also comparable to or smaller than τ_o, is small compared to τ_ℓ.

The conditions are quite different, though, for fcc alloys of the α-brass type, as well as in pure fcc metals after quenching or irradiation [64]. In these cases, the primary dislocations are assembled into bands of high density, parallel to the active slip planes. Dislocations on the secondary system can break through these only with difficulty, resulting in overshooting [18,52,53,85]. In fcc alloys, moreover, no appreciable mushrooming takes place, nor are many "jogs" formed or eliminated through dislocation uncertainty. Therefore the resolved shear stress in α-brass does not drop back to a much lower level once the second system has started to act, but the intersection jogs have permanently raised the level of the flow stress. The said difficulty for dislocations to intersect, as the main distinction between α-brass type alloys, one the one hand, and pure fcc metals (where dislocations intersect easily) on the other hand, was already pointed out and discussed in an earlier paper [38].

6) The Proportionality Between the Temperature Dependent and the Temperature Independent Part of the Flow Stress

In the recent literature much emphasis is put on experiments in which a metal is strained under certain testing conditions and then abrupt changes are made in temperature, speed of testing, or by repeatedly changing between, say, simple extension to twisting so that intersecting slip systems are activated [11,13,75,76,86-93]. Such experiments allow to determine what fraction of the observed flow stress is due to temperature independent processes (our τ_ℓ) and what fraction is due to thermally activated processes (τ_s), or they allow conclusions about the hardening due to intersecting dislocations. The results indicate (i) that additional dislocation intersections raise the flow stress in stage II by only a comparatively small amount,

(ii) that the rate of stage II hardening is unaffected by changes in the testing conditions once the specimen is strained beyond a certain transition stage, (iii) that in stage II up to about 10% or 20% of the flow stress is due to thermally activated processes, and (iv) that the temperature dependent part of the flow stress is linearly related to the whole flow stress, being almost proportional to it, not only in stage II but also in stage III, in single and in double glide and even after quenching [93]. The proportionality factor, linking the change in flow stress for a certain change in temperature or speed of testing to the flow stress before the change, does depend on temperature but not obviously on any other parameter. This experimental fact is called the Cottrell-Stokes law.

While points (i) to (iii) above are clearly in agreement with the present theory, point (iv) merits some discussion. A linear relationship of the type found would not be surprising in stage II, if it is considered that the temperature dependent processes which cause the linear hardening rate during easy glide persist also in stages II and III, and the consequent linear increase in τ_s is superimposed on stage II hardening. This is consistent with the point made in subsection 1) above, and is more or less the position taken in the theories followed by Seeger, Haasen and coworkers.

Adams and Cottrell [87], on the other hand, expressed the view that the dislocation arrangements stay similar to themselves, changing in scale only, and that this accounts for the proportionality between $\Delta\tau$ and τ .

Basinski [90], finally, stresses that $\Delta\tau$ and τ are proportional because both, τ_s and τ_ℓ are due to the same cause, namely the intersection between glide dislocations and forest dislocations. The temperature independent part of the flow stress, in his view, is due to the elastic repulsion between the intersecting dislocations as well as their partials, while the temperature dependent part is due to the actual intersections.

The first of the three approaches has much to commend itself, but it breaks down in stage III; as does the idea of Adams and Cottrell, since in stage III irregular tangles give way to a cell structure (see for example Swann[94]). The third theory has the defect that one cannot see how forest dislocations can cause so much hardening, and, indeed, alternating tensile and twisting experiments[11,75,76,86] show that the contribution to hardness due to intersecting dislocations in pure fcc metals is not large. Also a very strong orientation dependence of θ_{II}, and, presumably, a change in θ_{II} at the onset of double glide should be expected if "forest" hardening was really the dominant factor in workhardening.

The objections to the three ideas previously offered depend on the accuracy with which the Cottrell-Stokes law is obeyed even in stage III, and after quenching. Assuming that, for pure fcc metals, there is in fact no change at all in the proportionality constant linking $\Delta\tau$ and τ, during single as well as multiple glide, in stage II, in stage III and after quenching, then this must be taken as clear evidence that the temperature dependent and temperature independent parts of the flow stress are firstly more intimately connected than assumed by Seeger, and, secondly, that dislocations on primary and secondary systems cannot act materially differently, ruling out Basinski's approach.

In the framework of the present theory, the Cottrell-Stokes law can be understood for pure fcc metals, and in a modified form also for bcc metals. These are the substances in which dislocations tangle profusely. Here it is agreed that aside from cross slip and climb, which will be discussed in connection with stage III, (i) thermal activation cannot materially assist dislocations to break free from the obstacles presented by parallel or near-parallel dislocation segments, which is the foundation of the theory of stage II given in the present paper, and (ii) that dislocations which have to be intersected make a contribution to the temperature dependent part of the flow stress, (plus a small contribution to the temperature independent part, which, however, shall be neglected).

As long as the dislocations form three-dimensional tangles, made up of almost randomly oriented sections, (which may be isolated, or spread out along active slip planes, or may be assembled into cell walls), any segment of a spreading loop will meet roughly twice as many dislocations which are so steeply inclined to it that they are intersected, than links which are so nearly parallel that they block its progress. Hence, the temperature assisted intersections are always in a fixed ratio to the temperature independent blockings, i.e. the distance between intersections ~~are~~ is always in a fixed ratio to the mean dislocation length $\bar{\ell}$, which determines the temperature independent part of the flow stress. This is true only to the extent that tangles or cell walls can be taken to consist of randomly oriented dislocation sections, but it remains true, in first approximation, even for multiple slip, and thereby the Cottrell-Stokes law is explained.

An irregularity arises for small strains because there are other contributions to τ_s besides dislocation intersections, and the former are the more prominent the lower the strain.

The transition regions which are observed in all experiments of changing the testing conditions are not surprising: Firstly, the equilibrium distribution of jogs and of superjogs, due to dislocation intersections as well as mushrooming and the uncertainty of dislocation axes [45] must to some extent depend on crystal orientation, strain, strain rate and temperature. As a specimen is unloaded, the momentary dynamical equilibrium of superjogs and jogs is frozen in, but it reestablishes itself, though at a different level, after a small additional deformation under the changed testing conditions. Secondly, small rearrangements may take place even on unloading, and these are believed to be at least partly responsible for the so-called Haasen-Kelly effect [95]. Thirdly, some of the point defects generated during slip, and still unabsorbed, diffuse to and/or along the dislocations to lock them during ageing experiments. This last effect was shown to account for the recovery of damping and modulus changes after deformation by Granato, Hikata and Lücke [96] and gives rise to yield points after unloading, or ageing, or both [97,98].

Since the presence of dislocation tangles is held responsible for the Cottrell-Stokes law, this should also hold in bcc metals under such conditions that tangles are formed, except that some complications arise because presumably the Peierls-Nabarro stress as well as Cottrell locking contribute much more to τ_s than they do in pure fcc metals. In α-brass type alloys, on the other hand, no tangles are formed up to substantial strains and, therefore, the Cottrell-Stokes law should not be obeyed in them. Preliminary investigations on α-brass are in agreement with this conclusion [99].

Theoretical Considerations on Stage III Hardening

Following Seeger's theory [11,12] the onset of stage III is usually ascribed to the beginning of profuse cross slip, facilitated by stress and thermal activation.

Obviously, cross slip must reduce the parameter β in the present theory, since some fraction of the dislocation segments which are held up at attracting parallel or near-parallel dislocations can, by cross slip, coalesce with these, and either annihilate mutually or undergo a favorable dislocation reaction. At the same time, the parameter α is increased by cross slip because dislocation segments which would otherwise be held up by dislocation links repelling them, can circumvent these obstacles through cross slip.

Mader and Seeger [100] have pointed out that the preferential elimination of screw dislocations by cross lip explains the generation of deformation bands in stage III which was observed by them. Although their model is very specific, and does not correspond to experimental fact, their reasoning can be adapted to other dislocation arrangements.

As far as the theory of linear workhardening is concerned, no drastic changes are anticipated through the action of cross slip. Linear workhardening is expected to persist until or unless climb takes place, but the discussed decrease of the parameter β/α must lead to a corresponding decrease of the workhardening coefficient, which according to eq. 9 is given by $\theta \cong (G/8\pi)\ \beta/\alpha$.

In this connection it may be significant that stage III sometimes takes the form of a transition range, followed by almost linear hardening, which then gives way to a curved final section with continuously decreasing slope. The linear range in stage III was particularly noticed and investigated by Haasen [13] who comments that, θ_{III}, the coefficient of workhardening in the linear range of stage III, "depends on temperature in much the same manner" as θ_{II}; the ratio of $\theta_{II} : \theta_{III}$ being about 10 : 6 for nickel between 78°K and 300°K, and a little smaller, i.e. θ_{III} proportionately a little larger, at very low temperatures.

This result is most intriguing when taken together with the evidence presented in the early part of this paper that the value of $K = G/\theta_{II}$ for fcc metals is about 300 while that derived from polycrystalline iron and simple steels is 500. The ratio of the linear workhardening rate in ~~stage III~~ for metals gliding with profuse cross slip (iron) to those gliding without (fcc metals in stage II), hence, is about the same as the ratios of the workhardening rate in the linear portions in stages III and II of fcc metals, namely 6 : 10. From this it seems reasonable to conclude that the action of profuse cross slip reduces the value of the parameter β/α to about 6/10 its value for glide without cross slip, but does not otherwise affect the present theory of workhardening very greatly. The theory of linear hardening is therefore also applicable to bcc metals, and by inference to substances like AgCl which show pronounced pencil glide, but with the parameter β/α taken as $0.6/(3 \times 4) = 1/20$, more or less.

Beyond the linear range of stage III, if it is present at all, the workhardening rate drops further; down to zero in some cases. A reduction in τ_s cannot account for this, since τ_s is usually small compared to the flow stress in stage III to begin with. The only obvious remaining cause for the further reduced workhardening rate is dislocation climb*, respectively "conservative climb" [101]. This, coupled with slip and cross slip, allows any arbitrary dislocation motion, and thus allows the mutual annihilation of any type of dislocations, with a consequent reduction of the workhardening rate. Only this final part of the workhardening curve thus is due to "dynamical recovery".

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* Although, technically, mushrooming represents a type of climb it is not referred to as such in the present paper, since it is better understood as slip-induced precipitation of point defects [45].

It seems perfectly possible that dislocation climb may begin to operate extensively before cross slip. In that case no linear range is expected to occur in stage III. Which of the two mechanisms, climb or cross slip, begins to operate first, depends on several factors. In bcc metals cross slip will almost invariably begin before climb, and the opposite is true for hcp metals. In fcc metals the ease of cross slip is primarily a function of the stacking fault energy, as first pointed out by Schoeck and Seeger [102], while climb is primarily a function of testing temperature relative to the melting point. For this reason, care must be taken in experiments designed to evaluate the stacking fault energy from the stress at which stage III begins, since, sometimes, the onset of stage III indicates not the beginning of cross slip but of climb.

Summary

(i) Easy glide is explained as the range during which the specimen is gradually filled with dislocations. Non-uniform stress distribution in this region may give the appearance of workhardening. The true workhardening component in easy glide is primarily due to the accumulation on the dislocations of jogs, and the interactions between point defects and dislocations. The resultant stress increase contributes to the temperature dependent part of the flow stress, τ_s.

(ii) Since the stress during easy glide does not change much, and since it is this which determines the dislocation density, the extent of easy glide is largely determined by the average path of the dislocations in this first stage. The dependence of the extent of easy glide on orientation, temperature, impurity content and alloying can be simply explained on this basis.

(iii) The end of easy glide is reached when a quasi-uniform dislocation distribution has been established in the specimen. It is not directly connected with the beginning of double glide, but double glide leads to the drastic reduction of mean free dislocation paths, so that the onset of multiple slip causes easy glide to end after only small additional strains.

(iv) Experimental evidence shows that the dislocation arrangements during and after easy glide differ widely for different materials and testing conditions, even for specimens which subsequently deform with similar values of $K = G/\theta_{II}$, where G is the modulus of rigidity and θ_{II} is the workhardening coefficient in stage II. It seems therefore futile to build theories of stage II on models employing specific dislocation arrangements. Consequently, a theory of stage II hardening is developed which rests on a few basic principles only.

(v) Three fundamental assumptions are made. Firstly, that during stage II a quasi-uniform dislocation distribution exists which remains similar to itself and changes in scale only. Secondly, that the average dislocation density ρ is connected to $\bar{\ell}$, the average length of the dislocation segments which move coherently, as $\rho = m/\bar{\ell}^2$. Thirdly, that workhardening in stage II is mainly the result of increasing dislocation density and the resultant decrease in $\bar{\ell}$, to which the temperature independent part of the flow stress is connected as $\tau_{\ell} \approx Gb/\pi\bar{\ell}$.

(vi) With the above three assumptions, the expression $K \approx \pi mc\alpha/\beta \approx 8\pi\alpha/\beta$ is derived. In it c is a geometrical factor not far from unity, and taken as 1.5, α is the reciprocal of the fraction of all encounters between parts of of expanding dislocation loops and stationary dislocations which block their further movement; and β is the fraction of the total length of a newly expanding loop which remains trapped, adding to the dislocation content.

(vii) It is argued that only dislocations which are nearly parallel can block each others progress, yielding $\alpha \approx 3$. Further it is deduced that β should be about 1/4, since certain loop parts will coalesce or almost coalesce with dislocations of opposite sign, and thus not only vanish themselves but in addition remove the dislocations which they encountered. As a result $K = G/\theta_{II}$ becomes equal to about 100 π, in good agreement with the value observed for pure fcc metals.

(viii) Since the expression for K does not contain the Burgers vector, the theory is applicable to crystals in which the dislocations from pile-ups, to the extent that the pile-ups can be regarded as super dislocations.

(ix) Additional dislocation pinning adds to τ_s if it is temperature dependent, but does not change the value of K as long as the pinning remains constant. If the number of additional pinning points or their strength depend linearly on the shear strain, the linear workhardening law is still conserved, but with a changed value of θ_{II}. These results are applicable to any kind of pinning, be it due to jogs, precipitates, point defect or impurity locking, or "mushrooming".

(x) The theory predicts that only up to about one sixth of the work done on a specimen in the course of deformation in stage II is retained as stored energy.

(xi) In stage II the average slip line length is found as $\Lambda \approx [n/(\gamma - \gamma_0)] \times 10^{-4}$ cm if γ denotes the shear, γ_0 the shear at the end of easy glide, and n the number of dislocations moving together in a pile-up. This agrees well with measurements on copper and aluminum if n is taken equal to 5 and 1 respectively, and is considered to be a satisfactory result in view of the fact that direct observations in thin foils of pure fcc metals have never revealed any pronounced dislocation pile-ups.

(xii) The dislocation density in stage II is given by about $\rho \approx 5\times10^{11}(\gamma-\gamma_0)^2$.

(xiii) It is difficult to decide in which direction θ_{II} should be changed through multiple slip, and it is therefore concluded that the change cannot be drastic. This is in agreement with experimental fact.

(xiv) The Cottrell-Stokes law is explained for all crystals which form dislocation tangles composed of almost randomly oriented dislocation segments. In such cases the moving dislocations encounter roughly twice as many dislocations which they intersect as those which are almost parallel, and block their progress. In other words, the density of "forest" dislocations remains proportional to the total dislocation density, i.e. the average distance between forest dislocations remains proportional to the average coherently moving dislocation length $\bar{l}$. As long as the temperature independent part of the flow stress τ_l, is inversely proportional to $\bar{l}$, with the same proportionality factor, and as long as the temperature dependent part τ_s is mainly due to dislocation intersections, the Cottrell-Stokes law is obeyed whenever, the dislocations form tangles composed of almost randomly oriented segments; independent of whether the tangles fill the specimen almost uniformly, or are aligned along slip planes, or form cell walls, or are distributed in any other way.

(xv) Alloys of the α-brass type do not form tangles and should therefore not obey the Cottrell-Stokes law. This law must also break down in easy glide, when those temperature dependent contributions to τ_s which are not due to dislocation intersections become important while τ_ℓ becomes small.

(xvi) Transition effects upon the changing of testing conditions, on unloading and reloading, or intermediate ageing, can be accounted for.

(xvii) The profuse operation of cross slip is believed to lower the value of β/α, and with it the value of the workhardening coefficient, by about 40%, but to leave the linear law of workhardening in operation. This conclusion is substantiated by the fact that the workhardening rate during linear hardening of polycrystalline iron and steel, and that observed in the linear part of stage III in nickel are about the same, namely 60% of the value normally observed in stage II for fcc pure metals.

(xviii) A further reduction of the workhardening rate in stage III can be understood as the consequence of dislocation climb, respectively "conservative climb".

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Legend

Fig. 1 - Typical workhardening curve of fcc metals.

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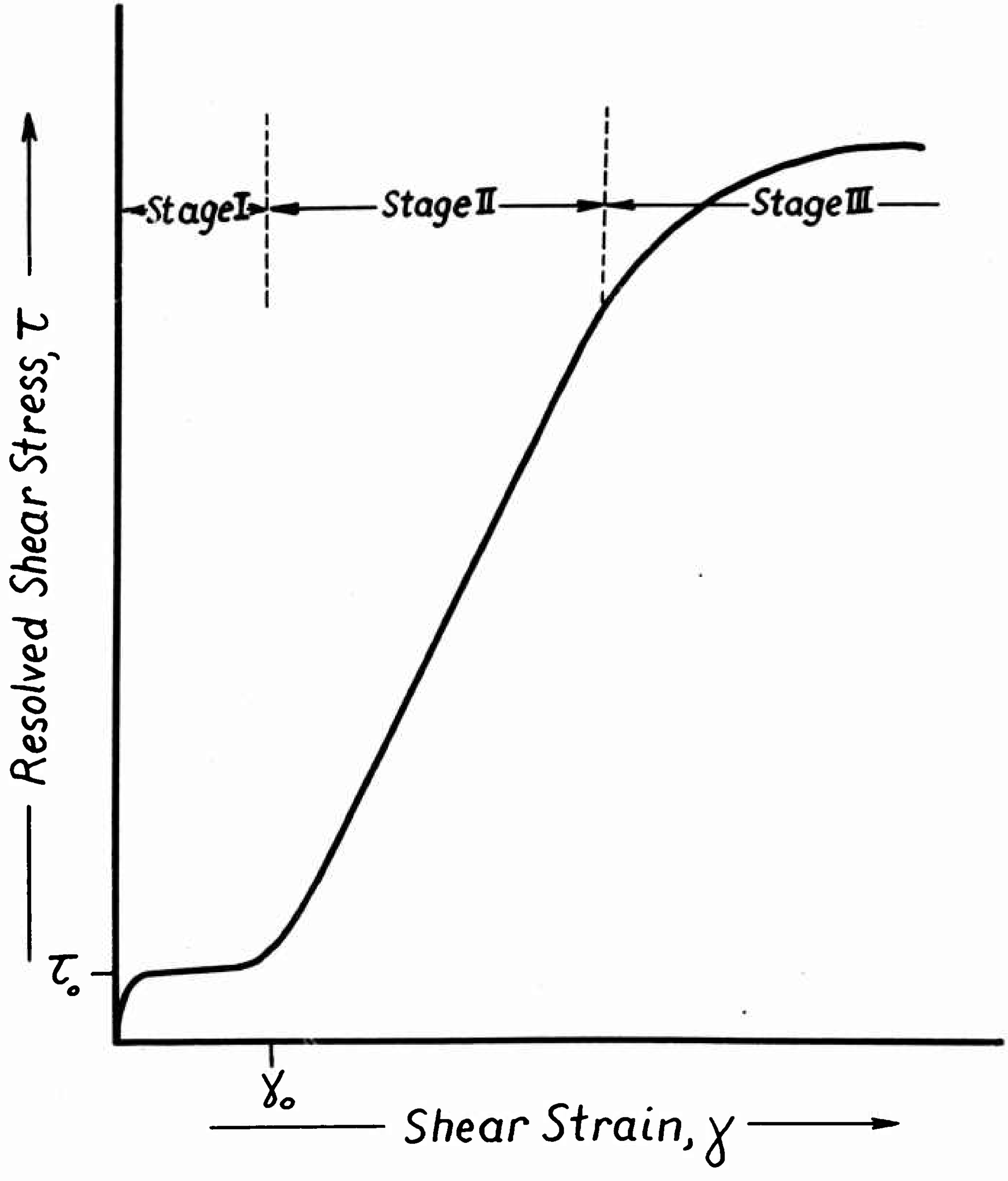
Fig. 1



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